Foundations of Semantics V: Intensionality

Rick Nouwen | R.W.F.Nouwen@uu.nl

February/March 2011, Gent

---

1 Extensionality and Interchangeability

(1) Substitution in formulae:
If ψ is a predicate logical formula, then $[\chi'/\chi]\psi$ is exactly like ψ, except that occurrences of χ are replaced by χ'.

(2) Principle of extensionality:
$\chi \leftrightarrow \chi' \quad \models \quad \psi \leftrightarrow [\chi'/\chi]\psi$

(3) Natural language counterparts:
a. "Ann is a woman and Bill a man" ↔ "Bill is a man and Ann a woman"
$\models$
"It is not the case that Ann is a woman and Bill a man" ↔ "It is not the case that Bill is a man and Ann a woman"
b. "My glass is half full" ↔ "My glass is half empty"
$\models$
"I know that my glass is half full" ↔ "I know that my glass is half full"

Gottfried Leibniz (1646–1716): Two things are identical if one can substitute the one for the other without loss of truth. (Eadem sunt quorum unum potest substitui alteri salva veritate)

(4) Identity in predicate logic:
$[\![t = t']\!]^{w,g} = 1$ iff $[\![t]\!]^{w,g} = [\![t']\!]^{w,g}$

(5) Leibniz's law is valid in predicate logic:
$s = t \quad \models \quad \varphi \leftrightarrow [t/s]\varphi$

Problem: the prinicple of extensionality / Leibniz's law is invalid for natural language

2 Non-extentionality in natural language

2.1. Frege's *morning star/evening star* paradox

(6) The facts:*Ancient Egyptians and Greeks had already discovered Venus, but thought it to be two separate heavenly bodies: the morning star (visible in the morning, also* Phosporos*) and the evening star (visible in the evening, also* Hesperos*). It was later discovered by the Greeks that the two were two occurrences of the same object.*

(7) a. The morning star is the morning star — true by form
b. The morning star is the evening star — a discovery

(8) The problem in predicate logic:

Since both morning star and evening star refer to the same object in the domain, and given Leibniz's law, we expect (7-a) and (7-b) to be equivalent

2.2. Intensional constructions

(9) a. John wants to talk to Joseph Ratzinger
b. John wants to talk to Pope Benedict XVI

(10) a. John believes that Barack Obama is the president of the United States of America
b. John believes that Barack Obama is the supreme commander of the U.S. armed forces

3 Sense and Reference

(11) In the world we live in: the reference of the morning star equals the reference of the evening star (equals the reference of venus).

(12) In Frege's theory, reference is, however, not determined directly. It is determined via *sense*.

(13) Sense (*mode of presentation / die Art des Gegebenseins*) — The following expressions have different senses but happen to have the same reference in our world
a. morning star
b. evening star
c. venus

(14) Similarly:
a. Barack Obama, the president of the US, the supreme commander of the US armed forces
b. the capitol of the Netherlands, the birthplace of Spinoza, Amsterdam

(15) The reference can be unknown (but the sense is there)
a. the winner of the lottery on January 1st 2020.
b. the president of the US in 2020

(16) Solving the paradox:
a. "The morning star is the morning star" is true independent of the reference. If you don't know the reference, you still know that this sentence is true
b. "The morning star is the evening star" is dependent on the reference.

4 Intension and Extension

(17) Assume that there are (infinitely) many possible worlds, such as w, w', w'', etc. as well as the world we live in, @.

(18) "The president of the United States"

extension in @: (the individual) Barack Obama ~ reference
intension: Barack Obama in @, Hilary Clinton in w, George Bush in w_{1992}, etc. ~ sense
(the intension is a function from worlds to entities)

(19) "MA student"

extension in @
extension in w''
extension in w_{2015}
the intension is a function from worlds to sets

(20) "admire"

extension differs across worlds
the intension is a function from worlds to relations between individuals

5 Opacity

A context is *opaque* if it gives rise to violation of the extensionality principle.

(21) Quotation
a. Peter said 'Joseph Ratzinger is smart'
b. ⇏ Peter said 'The pope is smart'

(22) Indirect speech
a. Peter said that Joseph Ratzinger is smart
b. ⇏ Peter said that the pope is smart

(23) Propositional attitudes (*know, believe, suspect* etc.)
a. Peter believes that Joseph Ratzinger is smart
b. ⇏ Peter believes that the pope is smart

(24) Intentional (with a 't') verbs (*look for, wish* etc.)
a. Peter is looking for Joseph Ratzinger
b. ⇏ Peter is looking for the pope

(25) Temporal designation
a. In 1985, the pope was Polish
b. ⇏ In 1985, Joseph Ratzinger was Polish

(26) Modals
a. The pope must be catholic
b. ⇏ Joseph Ratzinger must be catholic

(27) Modals
a. It's necessary that the pope is the pope
b. ⇏ It's necessary that the pope is Joseph Ratzinger

5.1. Does opacity always entail non-extensionality?

(28) Imagine Peter has just told me about his endless admiration of the theological work of the scholar Joseph Ratzinger. I then say to you:

(Although, he doesn't realise it,) Peter thinks that the pope is smart.

(29) Peter believes that the pope is smart

a. Peter's beliefs are such that he believes that the person he believes to be pope is smart
de dicto
b. Let p be the pope in the actual world. Peter's belief are such that he believes that p is smart
de re

6 Proper names and definite descriptions

Let W be a set of worlds.

(30) The interpretation of individual constants (Intensional version)

Let t be an individual constant:
Choice 1: $[\![t]\!]^w$ is constant for any value of w — individuals
Choice 2: $[\![t]\!]^w$ varies for different values of w — individual concepts

(31) Choice 1 is unsuitable for definite descriptions

The intension of "the pope" would be a constant function
That is, the extension of "the pope" would be the same in all worlds

(32) Peter believes that the pope is smart

a. The extension of the pope in the actual world: the individual J. Ratzenberger
b. The extension of the pope in a world according to Peter's belief: someone else
— This is incompatible with treating definite descriptions as individuals
— Compatible with individual concepts

Interpretation of *believe*

(33) Peter believes S

is true if and only if for all w such that w is compatible with Peter's beliefs it is the case that $[\![S]\!]^w$ is true.

(34) Peter believes the pope is smart

is true if and only if for all w such that w is compatible with Peter's beliefs it is the case that $[\![\text{the pope}]\!]^w$ has the property $[\![\text{smart}]\!]^w$

(35) w_1  w_2 

(36) "The card on the left"

a. ace of spades in w_1
b. king of spades in w_2

(37) Peter believes that the card on the left is the ace of spades.

$\Rightarrow$ w_2 is not compatible with Peter's beliefs

6.1. Proper names as rigid designators

(38) Kripke 1972:

a. definite descriptions: reference changes from world to world
b. proper names: reference is constant across worlds

To use Kripke's ideas in predicate logic, we could assume that a proper name corresponds directly to an individual constant which is interpreted rigidly across worlds. So, *John* is interpreted as j in every world. In contrast, we could assume that descriptions depend on the particular extension of a predicate constant in a particular world. We can do this using the *iota*-operator. This operator takes a proposition and returns an individual. For instance, $[\![\iota x.man(x) \wedge lazy(x)]\!]^{w,g}$ returns the unique individual who is both a man and lazy in world w. In other words, if we use $\iota x.man(x) \wedge lazy(x)$ as the semantics for the description *the lazy man*, we immediately predict that the description is not rigid, since the extension of 'lazy' and 'man' changes from world to world.

The following examples are from a paper by Bart Geurts (1997). Discuss.

(39) a. The Netherlands
b. The Mississippi
c. The Holy Spirit
d. The San Francisco Chronicle

(40) a. If I can choose between a blue car and a red car, I'll take the red car.
b. If a child is christened 'Bambi', then Disney will sue Bambi's parents.

7 Possible world semantics and Modal Predicate Logic

7.1. Possible world semantics

(41) Example.

a. $[\![president]\!]^{w}$ = {clinton}
b. $[\![president]\!]^{w'}$ = {obama}
c. $[\![admire]\!]^{w}$ = {⟨obama,clinton⟩}
d. $[\![admire]\!]^{w'}$ = {⟨obama,stevie-wonder⟩}
e. $[\![o]\!]^{w} = [\![o]\!]^{w'}$ = obama
f. $[\![c]\!]^{w} = [\![c]\!]^{w'}$ = clinton
g. $[\![s]\!]^{w} = [\![s]\!]^{w'}$ = stevie-wonder

Note: $[\![\iota x.president(x)]\!]^{w} \neq [\![\iota x.president(x)]\!]^{w'}$

7.2. Modal predicate logic

The syntax of modal predicate logic is that of predicate logic, except that we add two propositional operators $\Box$ and $\Diamond$. So, the following are examples of formulae in modal predicate logic that do not exist in the original predicate logic.

(42) a. $\Diamond(admire(b,c))$
b. $\Box(\forall x[student(x) \rightarrow lazy(x)])$
c. $\Diamond\Box\Diamond\neg\Box(\exists y[student(x)] \vee \Box lazy(c))$

(43) Accessibility

R is a relation that relates a world w to all the worlds accessible from w.

(44) Modal semantics

$[\![\Box(\varphi)]\!]^w = 1$ iff all worlds $w' \in R(w)$ are such that $[\![\varphi]\!]^{w'}$
$[\![\Diamond(\varphi)]\!]^w = 1$ iff some world $w' \in R(w)$ is such that $[\![\varphi]\!]^{w'}$

(45) Say, R is the relation such that $w' \in R(w)$ iff w' is in accordance with the laws of w

$\Diamond(paytaxes(a))$ — Ann is allowed to pay taxes
$\Box(paytaxes(a))$ — Ann must pay taxes

(46) $\Diamond(paytaxes(a))$

In some accessible world, Ann pays taxes. That is, there is a world that is in accordance with law, such that Ann pays taxes. In other words, paying taxes in compatible with the law.

(47) $\Box(paytaxes(a))$

In every accessible world, Ann pays taxes. I.e., there is no world that is in accordance with law, such that Ann does not pay taxes. I.o.w., not paying taxes is incompatible with the law.

(48) Say, R is the relation such that $w' \in R(w)$ iff w' is conceived as being possible in w.

$\Diamond(shoot(a, c))$ — Ann might shoot Carl
$\Box(athome(c))$ — Carl must be at home

We normally consider *should* to correspond to $\Box$ and *can't* to $\neg\Diamond$. Given (44), we expect (49) to be a contradiction (why?). Can you explain why it is nevertheless a sensible sentence?

(49) Carl should marry Ann, but he can't.

8 More on modality in Natural Language: Kratzer 1981

8.1. Propositions and worlds

(50) A proposition is something that is true or false in a world

e.g. *the earth is flat* is true in some worlds, false in others

(51) A proposition is a set of worlds (the set of worlds that make it true)

e.g. *the earth is flat* refers to the set of worlds in which the earth is flat

(52) A world can be characterised by the set of propositions it makes true

e.g. the world we live in refers to {the earth is not flat, Rome is the capitol of Italy, . . . }

(53) Consistency: A set of propositions A is consistent if, and only if, there is a world where all propositions of A are true.

e.g. {the earth is flat, Rome is the capitol of Italy} is consistent
e.g. {the earth is flat, the earth is not flat} is inconsistent

(54) Compatibility: A proposition p is compatible with a set of propositions A if and only if $A \cup \{p\}$ is consistent

(55) Logical Consequence: A proposition follows from a set of propositions A if and only if p is true in all worlds where all propositions of A are true.

8.2. Conversational Background and Modal Force

In modal predicate logic we used a relation R for accessibility to express different kinds of usages for $\Box$ and $\Diamond$. Kratzer proposes to use a notion of *conversational background* $f(w)$, the set of all propositions that are relevant to the modality in w.

(56) Two modal forces
   a. Necessity: A proposition is necessary in a world w with respect to a conversational background f if and only if it follows from $f(w)$.
   b. Possibility: A proposition is possible in a world w with respect to a conversational background f if and only if it is compatible with $f(w)$

Types of conversational backgrounds (this list is *not* exhaustive):

Realistic — the set of propositions that are true in the real world

Epistemic* — the set of propositions that are known to be true

Stereotypical — the set of propositions that follow the normal course of events

Deontic* — the set of propositions that are according to the law (or a set of rules)

Teleological* — the set of propositions that are such that a certain goal or aim is reached

Buletic* — the set of propositions that are such that a certain wish is granted

(57) a. Miss Marple considered her notes and concluded that *Carl might be the murderer*.
   b. *Dutch citizens must pay taxes.*
   c. To get good cheese, *you have to go to the Twijnstraat.*

(58) a. Necessity in English: *must, need to, have to, should, be required to*, etc.
   b. Possibility in English: *can, may, could, be allowed to, might*, etc.

(59) In view of what I know, Carl must be at home.
   Let f be a function that maps any world w to the set of propositions that (in w) I know to be true.
   E.g. $f(w)$ ={Carl's car is parked in front of his house, Carl never goes anywhere without his car, the lights are on in the house, etc.}
   (59) is true if and only if it follows from $f(w)$ that Carl is at home
   In other words, (59) is true if and only if Carl is at home in all worlds in which all the propositions in $f(w)$ are true

Exercises

(60) I need to sneeze!
   How would you analyse this example?

(61) Carl must be 21 years old.
   Which conversational backgrounds are compatible with this example and why do you think so?

(62) (According to the university laws,) every student should admire Carl.
   Analyse this sentence in modal predicate logic (discuss the nature of R)
   Analyse this sentence in Kratzer's approach