GIST – Gent
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Blandijn – Groote Vergaderzaal 3de verdieping - Blandijnberg 2 Gent

Quantification and Colour in Natural Language

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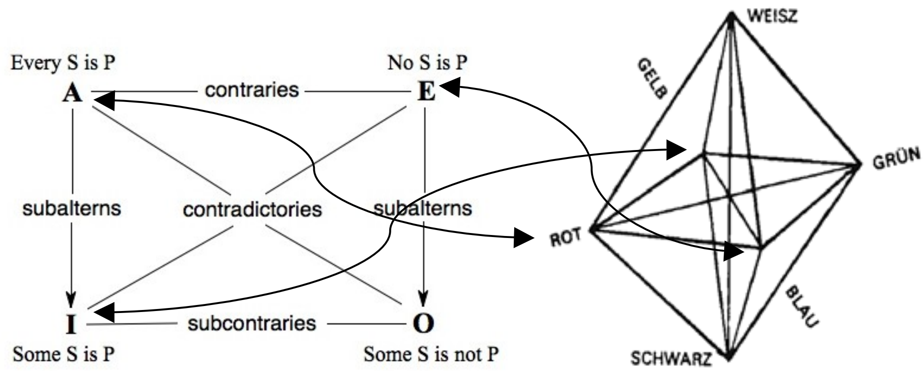
0. Introduction

(1) structure:

(1) definitions of oppositions in predicate logic, using a Smessaert-type bitstring-analysis with a string consisting of three values per quantifier.

(2) a mereological algebra of colours as an idealized binary basis for colour cognition.

(3) conclusion: isomorphism of the predicate logic and colour algebras



Square of Oppositions (Boethius, 5th century) Colour Octahedron (Höfler 1897)

(4) Natural language application: parallels between the two linguistic domains of application of the respective algebras:

- Evolution sequence of terms for quantifier and colour oppositions
- natural versus non-natural quantifiers (*nand/*nall) and colour terms (yellow vs. "cyan"): cognitive complexity and the generalized O-corner problem.

1. Predicate Logic (cf. Smessaert 2009)

(1)

U = {a,b,c,d,e,f,g,h}
D = {a,b,c,d} [[book]]
P1 = {e,f} [[be asleep]]
P2 = {c,d,g,h} [[be in English]]
P3 = {a,b,c,d,g,h} [[be worth reading]]
(Smessaert 2009: 304)

(2)

[ ]D:
$[x1 \ldots xn]D \equiv \{X \subseteq U : X \cap D = \{x1, \ldots, xn\}\}$

[a]D $\equiv$ {X $\subseteq$ U : X $\cap$D = {a}} [[the book a]]
[abc]D $\equiv$ {X $\subseteq$ U : X $\cap$D = {a,b,c}} [[the books a, b and c]]
[ ]D $\equiv$ {X $\subseteq$ U : X $\cap$D = $\varnothing$} [[neither a nor b nor c nor d]]

P1 $\in$ [ ]D [[neither book a nor b nor c nor d is asleep]]
P2 $\in$ [cd]D [[the books c and d are in English]]
P3 $\in$ [abcd]D [[the books a, b, c and d are worth reading]]"

(3)

D*n* $\equiv$ {X $\subseteq$ U :/ X $\cap$ D /= n}
D0 $\equiv$ [ ]*D*
D3 $\equiv$ [abc]*D* $\cup$ [abd]*D* $\cup$ [acd]*D* $\cup$ [bcd]*D*

(4) Scalar structure (partition of the Powerset of the Universe)

D4	D3	D2	D1	D0
[abcd]	[abc]	[ab]	[a]	[]
	[abd]	[ac]	[b]	
	[acd]	[ad]	[c]	
	[bcd]	[bc]	[d]	
		[bd]		
		[cd]		

(5)

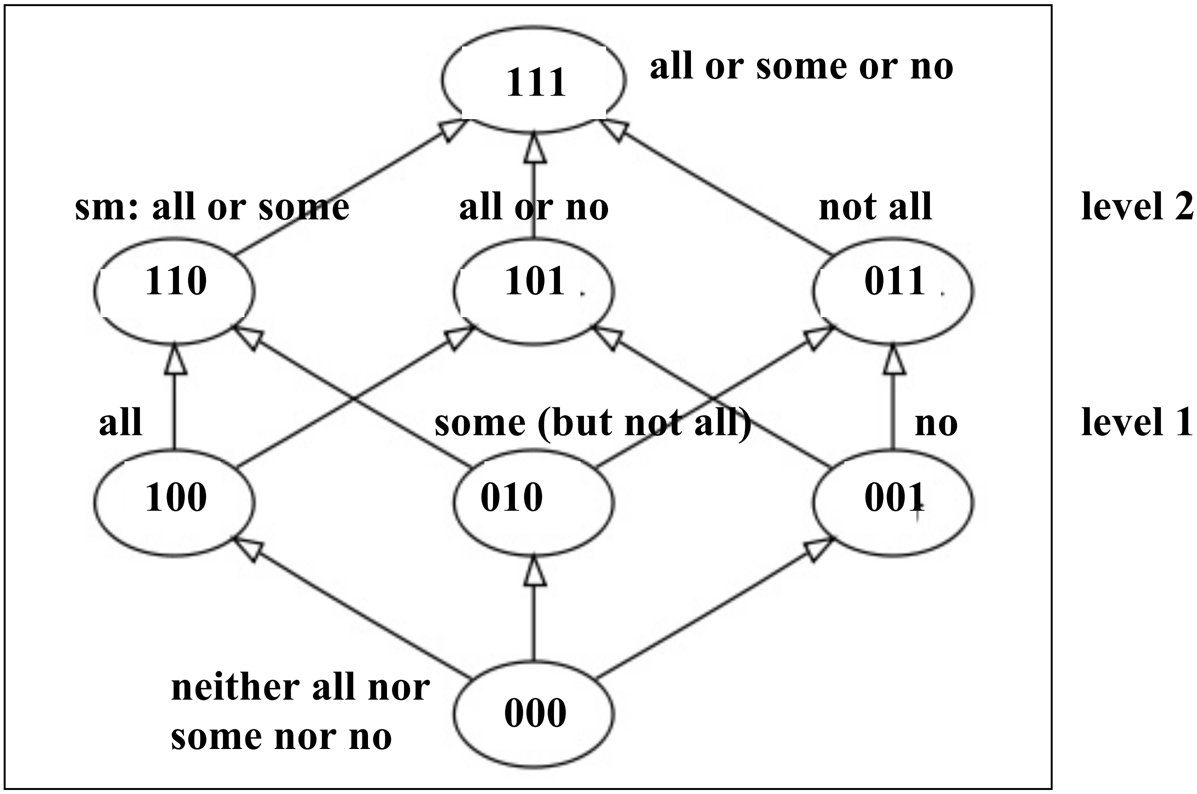
D4	D3	D2	D1	D0
		[ab]		
	[abc]	[ac]	[a]	
	[abd]	[ad]	[b]	
[abcd]	[acd]	[bc]	[c]	[ ]
	[bcd]	[bd]	[d]	
		[cd]		
Dmax	Dmax−1	D*n*	Dmin+1	**D**min
κ	-------------*λ* -------------			*μ*

(6) at the bottom of (5):

κ ≡ Dmax = {X ⊆ U :| X ∩ D |=| D |}
= {X ⊆ U : X ∩ D = D}
λ ≡ Dmin+1 ∪ … ∪ Dmax−1 = {X ⊆ U : | D | >| X ∩ D |> 0}
μ ≡ Dmin = {X ⊆ U :| X ∩ D |= 0}
= {X ⊆ U : X ∩ D = ∅}

"The bottom end *μ* denotes the set of all sets which do not intersect with the domain of quantification D, whereas the top end *κ* refers to the set of all sets which completely include the domain set D."

(7) Smessaert uses a shorthand notation format in the shape of a string of three bit positions. A value 1 for a particular area means that it is part of the quantifier denotation, whereas a value 0 indicates that it is not.



Hasse-diagram (Predicate Logic): 8 vertices

(8)

level 1 quantifiers (L1) $\kappa\lambda\mu$

A-corner[1]: [[all (books)]] $\equiv$ 100 = κ

Y-corner:[[some but not all (henceforth *some*) (books)]] $\equiv$ 010 = λ

E-corner: [[no (books)]] $\equiv$ 001 = μ

level 2 quantifiers (L2) $\kappa\lambda\mu$

I-corner: [[some (henceforth *sm*) (books)]] $\equiv$ 110 = $\kappa \cup \lambda$

U-corner: [[no or all (books)]] $\equiv$ 101 = $\kappa \cup \mu$

O-corner: [[not all (books)]] $\equiv$ 011 = $\lambda \cup \mu$

(9) Any L1 quantifier has a unique L2 quantifier as its contradictory and vice versa; any two L1 quantifiers are one another's contraries; and any two L2 quantifiers are one another's subcontraries. Here are the formal definitions in bit-string notation.

[1] The letters of the corners refer to their names in the Boethian Square of Oppositions and the Blanché (1969) star (cf. below).

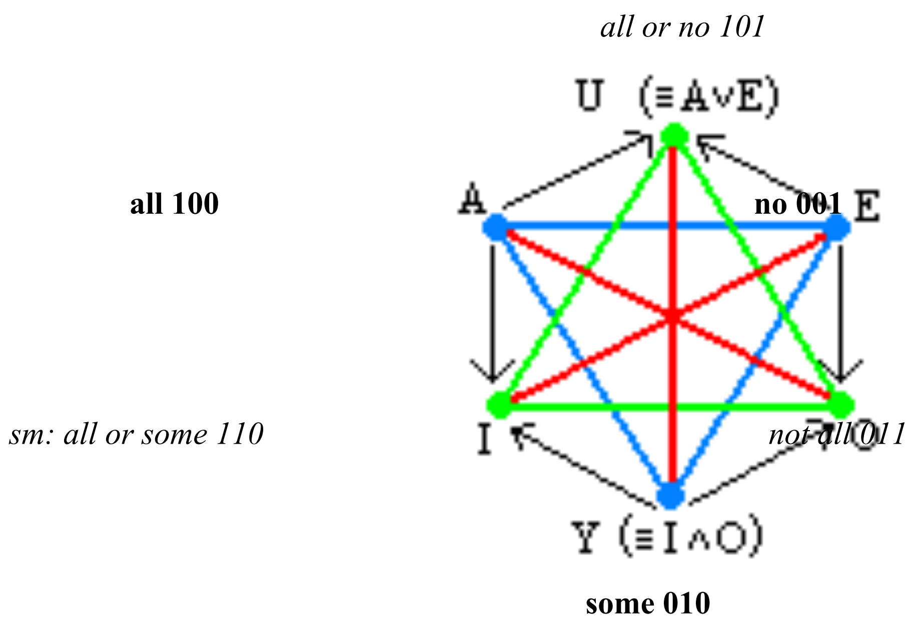
Contradictory(q1,q2) iff ([[q1]] $\cap$ [[q2]] = 000) *and* ([[q1]] $\cup$ [[q2]] = 111)
CD(no, sm) (001 $\cap$ 110 = 000) *and* (001 $\cup$ 110 = 111)
CD(all, not all) (100 $\cap$ 011 = 000) *and* (100 $\cup$ 011 = 111)
CD(some, no or all) (010 $\cap$ 101 = 000) *and* (010 $\cup$ 101 = 111)

Contrary(q1,q2) iff ([[q1]] $\cap$ [[q2]] = 000) *and* ([[q1]] $\cup$ [[q2]] $\neq$ 111)
CR(no, some) (001 $\cap$ 010 = 000) *and* (001 $\cup$ 010 $\neq$ 111)
CR(no, all) (001 $\cap$ 100 = 000) *and* (001 $\cup$ 100 $\neq$ 111)
CR(some, all) (010 $\cap$ 100 = 000) *and* (010 $\cup$ 100 $\neq$ 111)

Subcontrary(q1,q2) iff ([[q1]] $\cap$ [[q2]] $\neq$ 000) *and* ([[q1]] $\cup$ [[q2]] = 111)
SCR(not all, sm) (011 $\cap$ 110 $\neq$ 000) *and* (011 $\cup$ 110 = 111)
SCR(not all, all or no) (011 $\cap$ 101 $\neq$ 000) *and* (011 $\cup$ 101 = 111)
SCR(some, all or no) (110 $\cap$ 101 $\neq$ 000) *and* (110 $\cup$ 101 = 111)

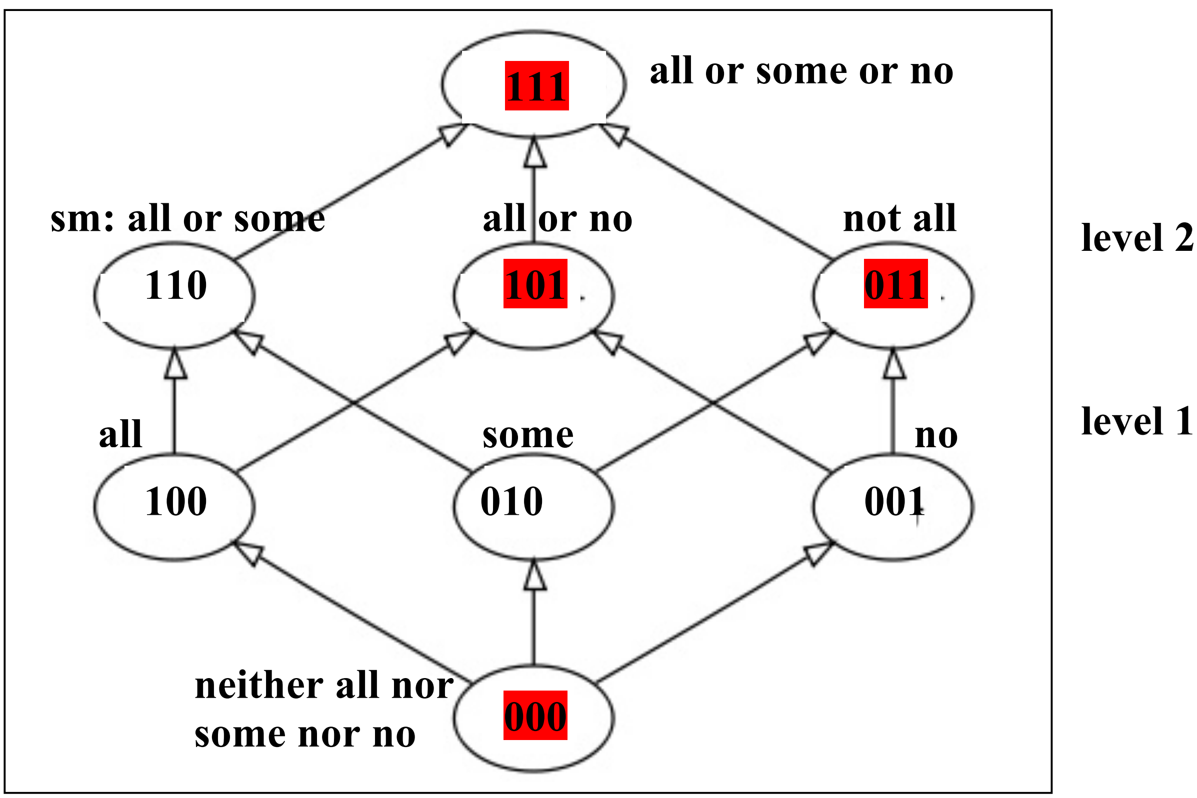
Entail(q1, q2) iff ([[q1]] $\cap$ [[q2]] = [[q1]]) *and* ([[q1]] $\cup$ [[q2]] = [[q2]])
iff [[q1]] $\subseteq$ [[q2]] (19)
Entailment
ENT(all, sm) 100 $\subseteq$ 110
ENT(all, no or all) 100 $\subseteq$ 101
ENT(some, sm) 010 $\subseteq$ 110
ENT(some, not all) 010 $\subseteq$ 011
ENT(no, not all) 001 $\subseteq$ 011
ENT(no, no or all) 001 $\subseteq$ 101

(10)
This setup can be represented by means of a bitriangular representation, a so-called Blanché-star (Blanché 1969), where contradictories are connected by red lines, contraries by blue lines and subcontraries by green lines.



Blanché star (Predicate Logic): 6 vertices (111 and 000 missing)

(11)
Looking at matters from a linguistic viewpoint, it is to be noted that the enriched representation by means of a Hasse diagram has four corners for which lexicalization by means of a single term is nonexistent or extremely rare, namely 111, 000, 011 and 101:



The fact that 000 and 111 do not lexicalize as a single word is because the predicates involved are trivial, i.e. completely non-informative and can therefore not serve for contingent situations (it is logically necessary that "all or some or no flags are green"). The other corners that do not lexicalize (with maybe a chance exception if Seuren is right) are both secondary operators whose intersection is the E-corner operator, which itself is the least often lexicalized of the level 1 corners cross-linguistically (cf. Horn 1989). Somehow, negative corners are less easily lexicalized or only non-naturally so (as in the case of the scientifically constructed O-corner item *nand*.

2. The mereological algebra of colours

(12) The Boolean definitions of the Aristotelian relations of Opposition straightforwardly carry over to the primary and secondary colours, modulo replacement of

(a) settheoretical union by mereological sum (⊕), the individuals involved in the operation being wavelengths of visible light (or alternatively activation of the cone in the retina that is sensitive to that particular wavelength). Hence, the mereological sum is the combination of the wavelengths of visible light in question, which yields a different colour. For example, the mereological sum of *RED* and *GREEN* yields *YELLOW*, which is indeed a combination of the wavelengths of visible light of *RED* and *GREEN*. Note the difference between such a mereological sum and settheoretical union: a description of the settheoretical union of *RED* and *GREEN* would be "*RED* or *GREEN* or the combination of *RED* and *GREEN* (i.e. *YELLOW*)". A mereological sum does not include the first two disjuncts (*RED*, *GREEN*) but only the combination, the reason being that mereology is interested in nontrivial part-whole relationships, and *YELLOW* is the only nontrivial holonym for the meronyms *RED* and *GREEN*.

(b) settheoretical intersection by mereological product (⊗), which amounts to reducing the individuals involved to the wavelength(s) of visible light they have in common. This is what happens when we mix colours, which amounts to removing or blocking the wavelengths that the colours one mixes do not share. For example, when we mix *YELLOW* (which is the mereological sum of the wavelengths of *RED* and *GREEN* as we saw above) and *MAGENTA* (which is the mereological sum of *RED* and *BLUE*), we end up with what they share: *RED*;

(c) quantifiers by mereological individuals, i.c. colours such as *RED*, *GREEN*, etc.;

(d) The settheoretical null set and universe by BOTTOM and TOP respectively, which are individuals in their own right. In the colour algebra they are respectively *BLACK* and *WHITE*. Note that there is often controversy about the status of a BOTTOM in a mereological system. Thus, one might argue in our case that *BLACK* is qualitatively different from all the rest in that it is really the *absence* of cone activity and therefore only an "individual" by reification of the absence of activation of any cone due to the absence of any wavelength of visible light into something. But we do see *BLACK* of course, so the idea that there is a BOTTOM is justified. The only problem that poses from the viewpoint of naturalness is that *BLACK* trivially a "part" of *RED* (and any other colour), parallel to the way in which the null set is a subset of any set.

So, let's have a look at the resulting algebra, which turns out to be perfectly isomorphic to the bitstring analysis for the quantifiers of predicate logic above:

(13)

level 1 (primary) colours (L1) $\kappa\lambda\mu$

A-corner: *RED* ≡ 100 = κ

Y-corner: *GREEN* ≡ 010 = λ

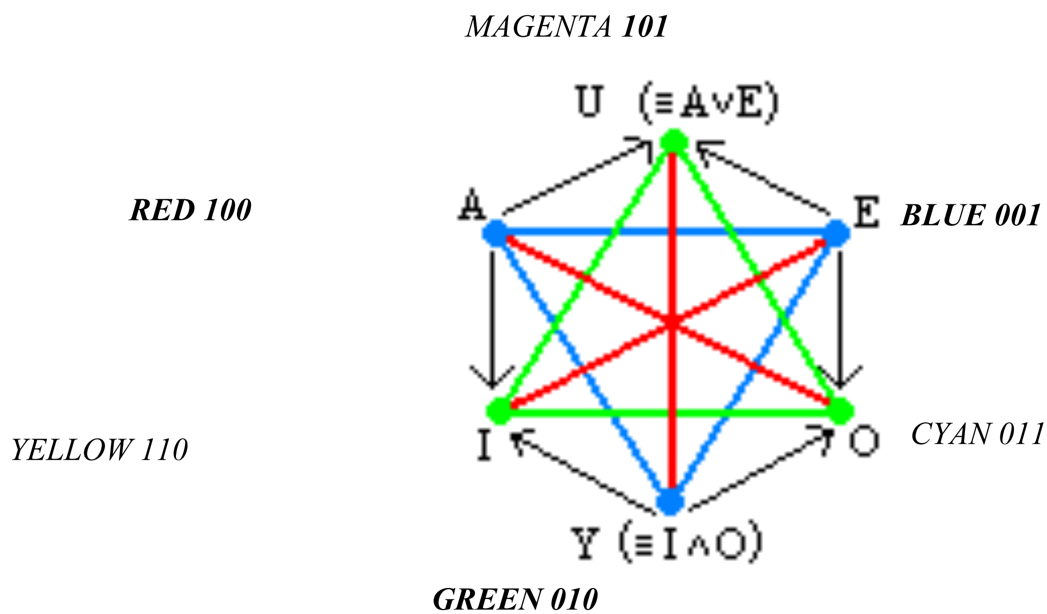
E-corner: *BLUE* ≡ 001 = μ

level 2 (secondary) colours (L2) $\kappa\lambda\mu$

I-corner: *YELLOW* ≡ 110 = $\kappa \oplus \lambda$

U-corner: *MAGENTA* ≡ 101 = $\kappa \oplus \mu$

O-corner: *CYAN* ≡ 011 = $\lambda \oplus \mu$



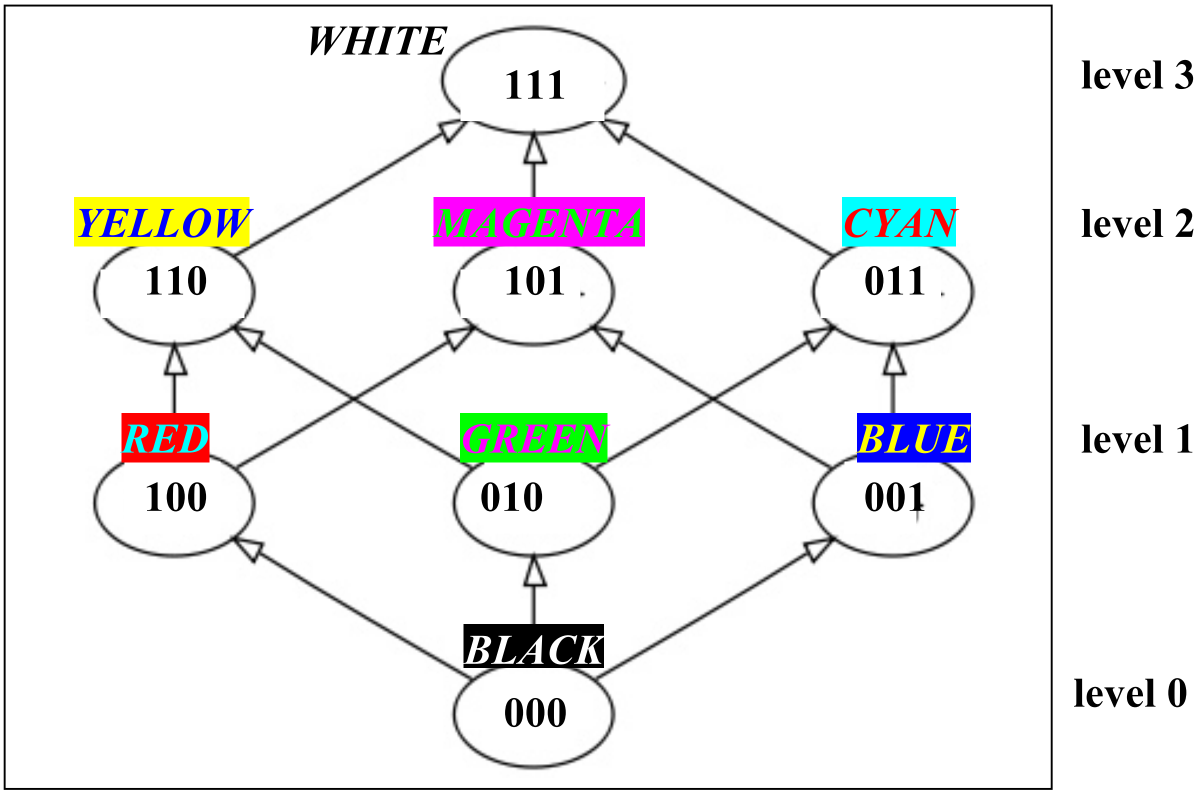
Blanché star (Colours): 6 vertices (*WHITE* (111) and *BLACK* (000) missing)

(14) two colours have to be added:

Level 0 (BOTTOM) colour (L0): *BLACK* ≡ 000 = $\kappa \otimes \lambda \otimes \mu$

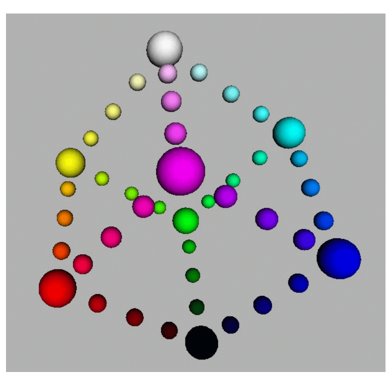
Level 3 (TOP) colour (L3): *WHITE* ≡ 111 = $\kappa \oplus \lambda \oplus \mu$

(15) A Hasse-diagram can easily accommodate these two new vertices.



Hasse-diagram (Colours): 8 vertices

(16) A three-dimensional version:



The colour cube (c/o Hans Smessaert)

(17)

Any L1 colour has a unique L2 colour as its negate or complementary (which is the mereological counterpart of *contradictory* in logic) and vice versa, any two L1 colours are one anothers "contraries" (= nonoverlapping colours whose mereological sum is not the mereological TOP colour WHITE (111)), and any two L2 colours are one anothers "subcontraries",i.e. mutually partially overlapping colours whose mereological sum is the mereological TOP colour WHITE.

Negate (or "complementary"): the negate of A, NEG(A), is that individual whose parts are exactly those that are discrete from A. The negate is the mereological counterpart of contradictoriness in logic. In the mereological algebra of colours the negate of a colour is what is known as its complementary colour.

Mereological "contradictories" = Negates = Complementary colours(COMP):

CD(c1,c2) iff ([[c1]] $\otimes$ [[c2]] = 000) *and* ([[c1]] $\oplus$ [[c2]] = 111)

CD(*BLUE, YELLOW*) (001 $\otimes$ 110 = 000) *and* (001 $\oplus$ 110 = 111)

CD(*RED, CYAN*) (100 $\otimes$ 011 = 000) *and* (100 $\oplus$ 011 = 111)

CD(*GREEN, MAGENTA*) (010 $\otimes$ 101 = 000) *and* (010 $\oplus$ 101 = 111)

CD(*BLACK, WHITE*) (000 $\otimes$ 111 = 000) *and* (000 $\oplus$ 111 = 111)

Mereological "contraries" (CR) = primary colours:

CR(c1,c2) iff ([[c1]] $\otimes$ [[c2]] = 000) *and* ([[c1]] $\oplus$ [[c2]] $\neq$ 111)

CR(*BLUE, GREEN*) (001 $\otimes$ 010 = 000) *and* (001 $\oplus$ 010 $\neq$ 111)

CR(*BLUE, RED*) (001 $\otimes$ 100 = 000) *and* (001 $\oplus$ 100 $\neq$ 111)

CR(*GREEN, RED*) (010 $\otimes$ 100 = 000) *and* (010 $\oplus$ 100 $\neq$ 111)

Mereological "subcontraries" (SCR) = secondary colours

SCR(c1,c2) iff ([[c1]] $\otimes$ [[c2]] $\neq$ 000) *and* ([[c1]] $\oplus$ [[c2]] = 111)

SCR(*CYAN, YELLOW*) (011 $\otimes$ 110 $\neq$ 000) *and* (011 $\oplus$ 110 = 111)

SCR(*CYAN, MAGENTA*) (011 $\otimes$ 101 $\neq$ 000) *and* (011 $\oplus$ 101 = 111)

SCR(*YELLOW, MAGENTA*) (110 $\otimes$ 101 $\neq$ 000) *and* (110 $\oplus$ 101 = 111)

Mereological "Entailment" = Proper parthood (PP)[2]

PP(c1, c2) iff ([[c1]] $\otimes$ [[c2]] = [[c1]]) *and* ([[c1]] $\oplus$ [[c2]] = [[c2]])

iff [[c1]] $\subseteq$ [[c2]]

Proper Parthood

PP(*RED, YELLOW*) 100 $\subset$ 110

PP(*RED, MAGENTA*) 100 $\subset$ 101

PP(*GREEN, YELLOW*) 010 $\subset$ 110

PP(*GREEN, CYAN*) 010 $\subset$ 011

PP(*BLUE, CYAN*) 001 $\subset$ 011

PP(*BLUE, MAGENTA*) 001 $\subset$ 101

PP(*RED, WHITE*) 100 $\subset$ 111

PP(*GREEN, WHITE*) 010 $\subset$ 111

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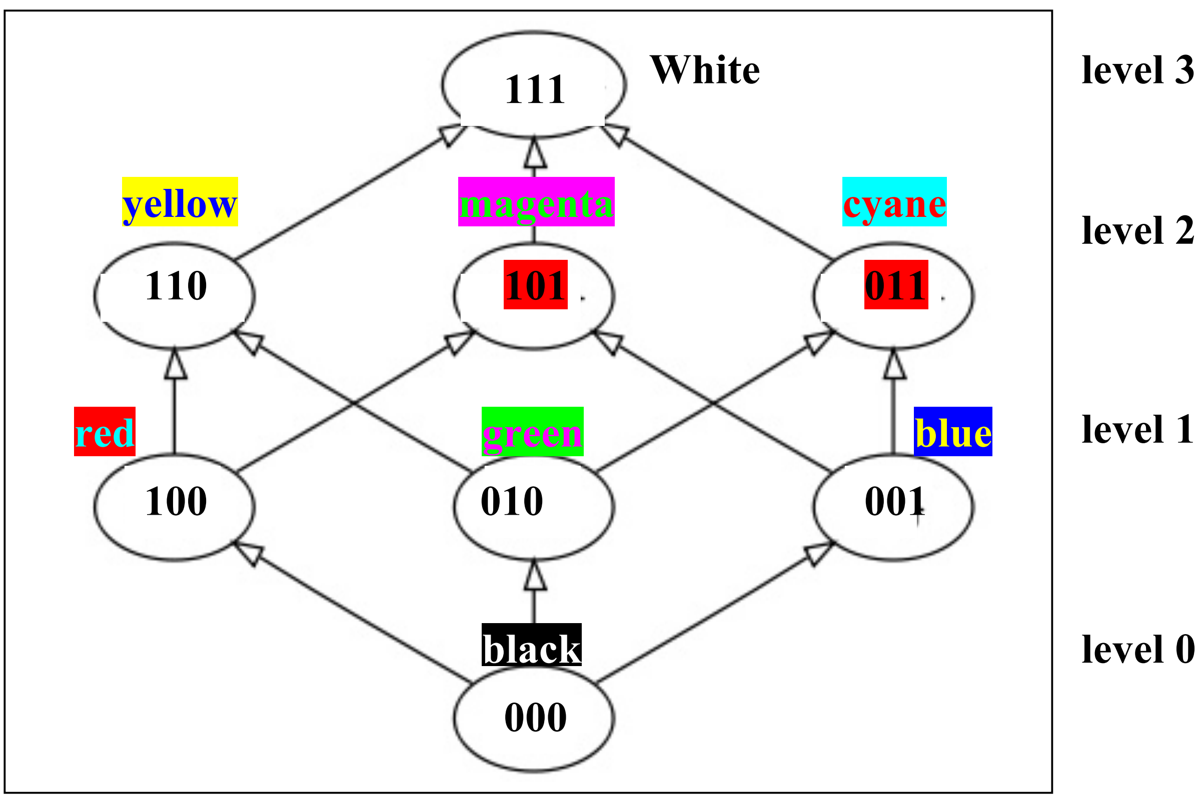
[2] Instead of using the equivalent of set inclusion, this version of mereology employs the equivalent of PROPER set inclusion (otherwise one could say that RED is a part of itself, which stretches our natural intuition of part–whole (meronym – holonym) relations). In this respect, this variant of mereology is like natural set theory in Seuren's sense.

PP(*BLUE, WHITE*) 001 $\subset$ 111
PP(*YELLOW, WHITE*) 110 $\subset$ 111
PP(*MAGENTA, WHITE*) 101 $\subset$ 111
PP(*CYAN, WHITE*) 011 $\subset$ 111

(18) Let us look at the linguistic side of the matter now and consider the status of the colour names in the different corners from the perspective of natural vs. non-natural (or alternatively nonexisting) lexicalization. In the colour algebra, the enriched representation by means of a Hasse diagram has only two corners for which lexicalization by means of a single term is not a basic natural colour term, namely 011 (cyan) and 101 (magenta), exactly the equivalents of the two level two corners which resisted lexicalization in the algebra for predicate logical operators. The other two that were not lexicalisable in the predicate calculus, are now the locuses of white (111) and black (000), respectively. This is a direct consequence of the difference between a mereological sum and set-theoretical union: whereas 111 denotes the whole universe in the case of predicate logic, it does not denote the whole universe of colours in a mereology (which would amount to "*RED* or *GREEN* or *BLUE*, etc.). Just as the mereological sum of *GREEN* and RED only denotes the combination of the wavelengths of *GREEN* and *RED* and not the two primaries that enter into it, white only denotes the mereological sum of chromatic colours, but none of those chromatic colours themselves. In that sense the mereological sum contains a combination of chromatic colours, but does not denote the colours that enter into it. The enriched diagram with two extra vertices beyond those available in the Blanché star is therefore crucial for the representation of the colour algebra. Not so for predicate logic, for reasons of noninformativity specified earlier.



Hasse-diagram (Colours): 8 vertices

3. Conclusion

The algebras of predicate logic and colours are perfectly isomorphic. The natural predicate logic of language has a cognitively deeper counterpart, the natural logic of colours.

4. Natural language application

4.1 natural versus non-natural quantifiers (*nand/*nall) and colour terms (yellow vs. “cyan”)

- Evolution sequence of natural terms for quantifier and colour oppositions

The incremental sequence for predicate logic operators as worked out in Jaspers (2005) along Peircean lines on the basis of the operator NEC is very similar to the Berlin-Kay (1969) incremental sequence for cross-linguistic colour term systems. They observed that if a language has three colour terms, they will always be dark/black, light/white and red; if another term comes in, it will be green or yellow, languages with one more colour will have the one they did not have yet (yellow or green) as the next one, and only languages who have yet another colour term will have blue, a term which many languages do not possess (cp. the relative infrequency of E-corner items noted by Horn 1989)

	1	2	3	4
NEC	> sm (or all)	> all	> some (but not all)	> no
“dark” (BLACK+BLUE)	“light” (YELLOW+WHITE+…)	“red” (RED)	“green” (GREEN)	“blue” (BLUE)
			“yellow” (YELLOW)	

4.2 cognitive complexity and the generalized O-corner problem

There are no natural names for colours 101(magenta) and 011 (cyan), just as there are no natural simplex names for quantifiers 101 (all or no) and 011 (*nand/*nall). General observation: the basic colours that attempt to incorporate 001 (BLUE/NO) have lexicalization trouble or are cognitively less accessible.

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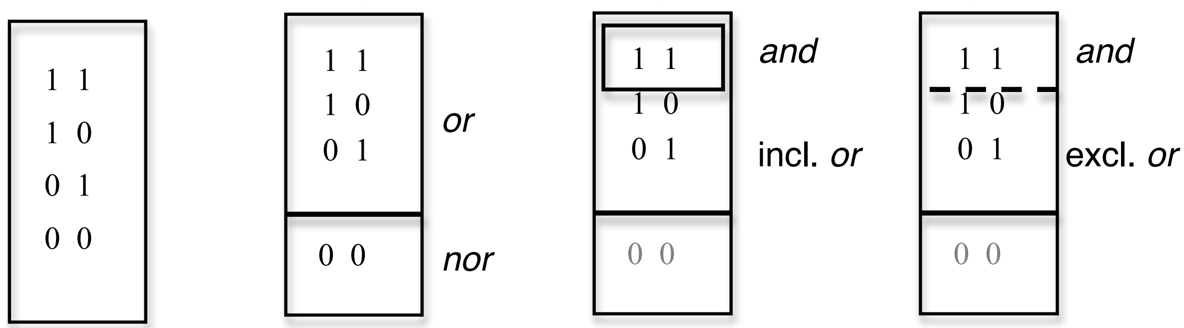
APPENDIX - SOME EXTENSIONS:
Extract from: R.K.Larson & D. Jaspers: "Broad and Narrow Language Faculties", Handout Workshop Evolution of Human Cognition, Georgetown Washington (18.03.2011)

1.3 Oppositional Structure (Jaspers 2005)

Jaspers (2005) offers a solution to the lexicalization questions. He derives logical concepts by making subtractions from a fixed domain space of values via a series of successive binary divisions. There is an initial exhaustive division between the contradictories Nor and Or (16a); within the remaining non-Nor space of values, we can either carve out the subset And, leaving inclusive Or as super set space (16b), or we can divide the inclusive Or space exclusively into And and exclusive Or (16c)

(16) a. *Domain* b. *Step 1* c. *Step 2* d. *Step 2'*



a. All truth-value pairs

b. **Contradiction**
Something is true
vs.
Nothing is true

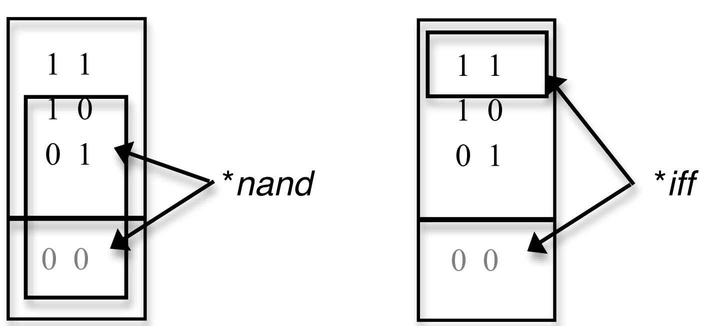
c. **Implication**
Something is true
vs.
Everything is true

d. Something but not everything is true
vs.
Everything is true

Natural logic terms correspond to concepts that match natural divisions of the concept space.

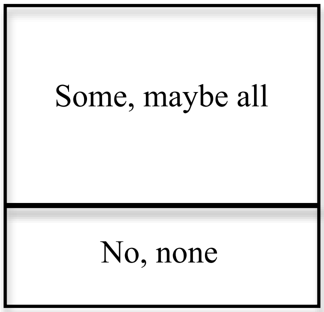
Non-naturalness with **nand* and **iff* follows from their concepts being non-congruent with natural divisions of the concept space; both cut across the basic Nor-Or division (26a,b):

(17) a. *Illicit Term* b. *Illicit Term*



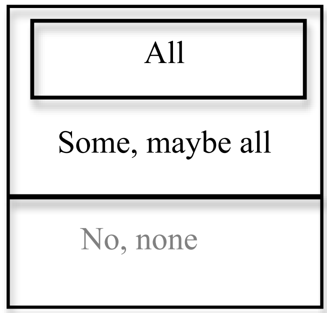
Exactly parallel considerations apply to the set of quantificational concepts and the terms for them (18a-18e):

(18) a. *Step 1*



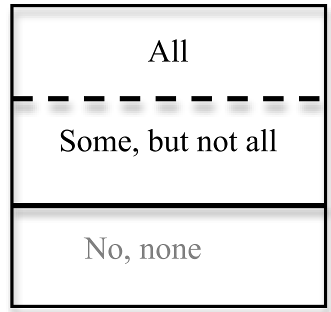
Contradiction
None vs. *Some,maybe all*

b. *Step 2*

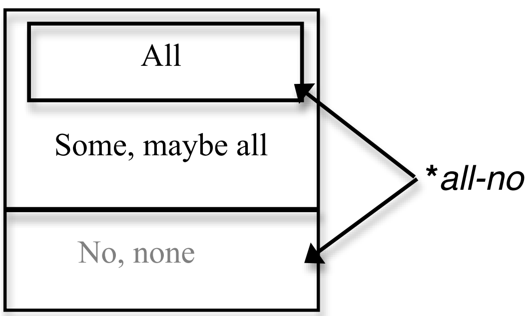


Implication
All vs. *Some,maybe all*

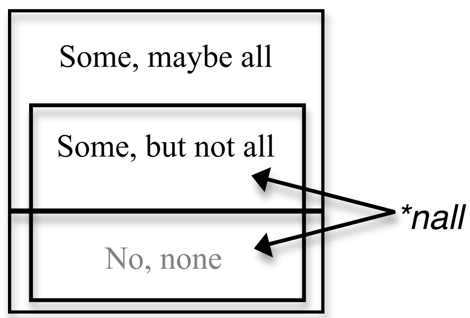
c. *Step 2'*



d. *Illicit Term*



e. *Illicit Term*



Summary:

Human cognition appears to deploy constraints on logical concept formation:

- Reflected in the availability of <u>naturally occurring</u> words for certain concepts, but not in availability *per se*. Non-natural concepts in the domain (*nand*, *nall*, *nnecessary*) can be lexified by system-external means, or by productive, compositional means. They are unavailable as "natural" atoms, but this mechanism structures only a subpart of the domain.
- The constraints are not on combinatorics of syntactic elements (not on "narrow syntax"), but rather on the systematic partioning of an antecedently given conceptual space. Crucial is the principle: divisions are made within existing divisions, and not across them. This accounts for "missing words".

2.0 The Logic of Color

Colors have long been thought to embody a "logic".

- The focal, achromatic color opposition black/white, representing absence/presence of all colors, appears analogous to false/true.

- Colors exhibit binary/ternary relationships reminiscent of logical ones.

Complementary colors: colors that, mixed in proper proportion, yield an achromatic/ neutral color (black/white/grey). Reminiscent of **contradiction**.
Primary colors: triads of colors that, mixed in proper proportion, yield the space of all chromatic colors.

Additive Primaries: colors whose **lights** combine to yield the space of all chromatic colors. E.g., Red-Green-Blue (RGB)

Subtractive Primaries: colors whose **pigments** combine yield the space of all chromatic colors. E.g., Cyan-Yellow- Magenta (CYM)

Additive primaries work together by **adding** lights of different wavelengths; combination of the three additive primaries yields **white**, the presence of all colors.

Subtractive primaries work together by **removing** lights of different wavelengths from the reflectance of an object; combination of the three subtractive primaries yields **black** (dark brown), the absence of all color.

The additive primaries have their complementaries in the subtractive primaries:

(18)

RBG		**CYM**
Red	–	Cyan
Blue	–	Yellow
Green	–	Magenta

Furthermore, subtractive primaries are (perceptual or non-perceptual) combinations of additive primaries:

(19) a.

$$\begin{tikzcd} & \text{Magenta (reddish-blue)} & \\ \text{Red} \arrow[ur] & & \text{Blue} \arrow[ul] \end{tikzcd}$$

b.

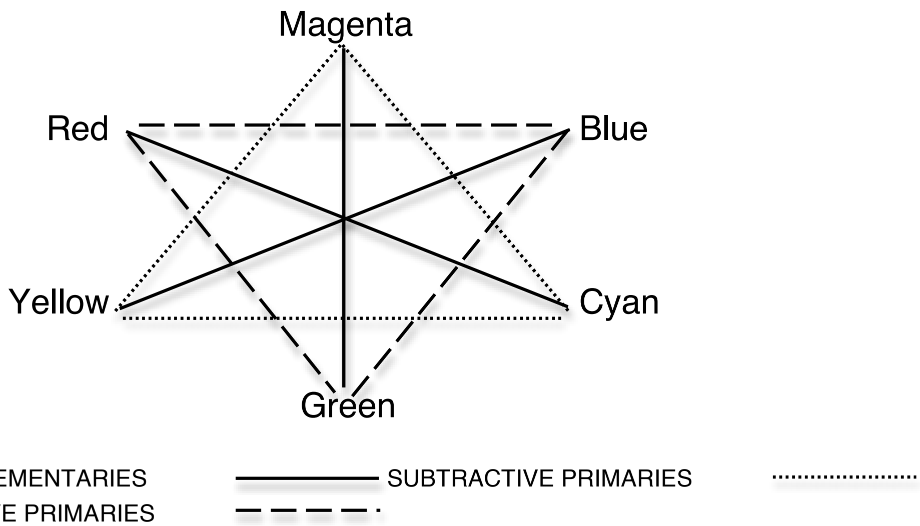
$$\begin{tikzcd} & \text{Cyan (blue-green)} & \\ \text{Blue} \arrow[ur] & & \text{Green} \arrow[ul] \end{tikzcd}$$

c.

$$\begin{tikzcd} & \text{Yellow} & \\ \text{Red} \arrow[ur] & & \text{Green} \arrow[ul] \end{tikzcd}$$

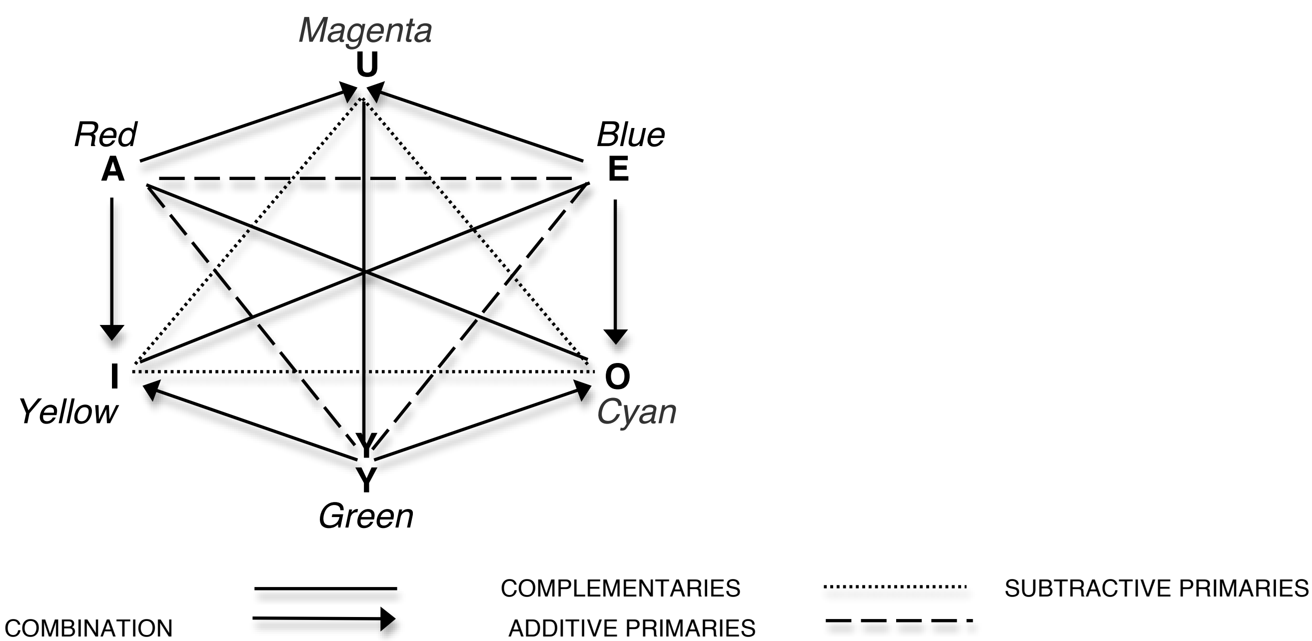
Following Jaspers (2011), we may arrange the two triangles of primaries with their relations of complementarity as in (20):

(20)



Adding relations of combination derives the hexagon (21):

(21)



The corner labeling is intended to draw out lexicalization parallelisms between this figure, and (10)/(14) for the logical relations.

- Of the six color terms present in the hexagon, only four are felt as “natural”: RBYG.
- The two non-natural words (CM) occupy the O and U corners

The correctness of this mapping is further supported by the important status of yellow.

2.1 The Special Status of Yellow

(21) reveals an asymmetry between CM vs. Y:

- CM are **perceptual combinations** of their contributing additive primaries (RB/BG, resp.).
- Y is **not a perceptual combination** of RG; i.e., Y is not perceived as reddish-green or greenish-red, but as a distinct, unary color.
- Y seems to enter into its own perceptual combinations with RG:
  Y + R = Orange; Y + G = Yellow-Green

Y's apparent status as a unary color led to vigorous debate about the true identity of the additive primaries.

2.1.1 Is Yellow a Primary?

RGBers: RGB is the true triad: R corresponds to activation of the long wavelength sensitive (L) retinal cone cells, G corresponds to activation of the medium wavelength sensitive (M) cone cells, and B corresponds to activation of the short wavelength sensitive (S) cone cells.
RYBers: RYB is the true triad: R corresponds to activation of (L) cones, Y corresponds to activation of L+M cones, and B corresponds to activation of (S) cones.
Jaspers (2011) notes that this debate strongly parallels those in logic regarding the status of middle terms (*or*, *some*) - whether they are **exclusive** or **inclusive**:

(22) *Competing Primary Colors*

RGBers argue for an exclusive middle color; RYBers argue for an inclusive one.

<table>
<tr><th>Percept Source</th><th>RGB Model</th><th colspan="2">RYB Model</th></tr>
<tr><td>L-cone</td><td>Red</td><td>Red</td><td rowspan="2">Yellow</td></tr>
<tr><td>M-cone</td><td>Green</td><td></td></tr>
<tr><td>S-cone</td><td>Blue</td><td colspan="2">Blue</td></tr>
</table>

(23) *Competing Logical Models*

<table>
<tr><th>Concept Type</th><th>Exclusive Some</th><th colspan="2">Inclusive Some</th></tr>
<tr><td>Universal affirmative</td><td>All</td><td>All</td><td rowspan="2">Some, maybe all</td></tr>
<tr><td>Particular affirmative</td><td>Some, but not all</td><td></td></tr>
<tr><td>Universal negative</td><td>No/None</td><td colspan="2">No/None</td></tr>
</table>

<table>
<tr><th>Concept Type</th><th>Exclusive Or</th><th colspan="2">Inclusive Or</th></tr>
<tr><td>Conjoined affirmative</td><td>And</td><td>And</td><td rowspan="2">either, maybe both</td></tr>
<tr><td>Disjoined affirmative</td><td>either, but not both</td><td></td></tr>
<tr><td>Conjoined negative</td><td>Nor</td><td colspan="2"></td></tr>
</table>

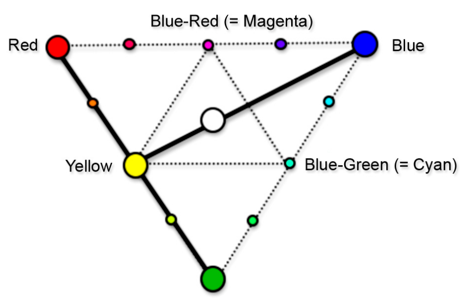
2.1.2 Binary Color Oppositions

Hering (1964/1920) proposes the RGB base has supermiposed on it **a pair of binary oppositions that do not yield mixed colors**, B – Y and R – G:

B + Y = B when S-cone activation is dominant (lighter B = more L+M, but S still dominant);
W when L+M = S;
Y when L+M-cone activation is dominant (lighter Y = more S, but L+M still dominant).

R + G = R when L-cone activation is maximal, M-cone activation zero
Orange as M-cone activation increases, but L > M
Y when L = M
YG as L-cone activation decreases from maximum, and L < M
G when M-cone activation is maximal, L-cone activation zero

(24)



Hering's proposal has support from comparative/evolutionary studies of primate color vision.

- Dichromactic color vision is the historic and contemporary mammalian norm (Jacobs 2009a, 2009b).
- Trichromacy appears to have arisen in primates from a dichromat state by development of a novel M/L photopigment (Jacobs 2009b) – i.e., elaboration at the yellow pole.
- Ability to discriminate red-green may have been selectively advantageous in distinguishing fruits from a foliage background (Jacobs 2005), or in distinguishing immature, easily digested, protein-rich foliage (which often flushes red in the tropics) from mature green foliage (Dominy and Lucas 2001).

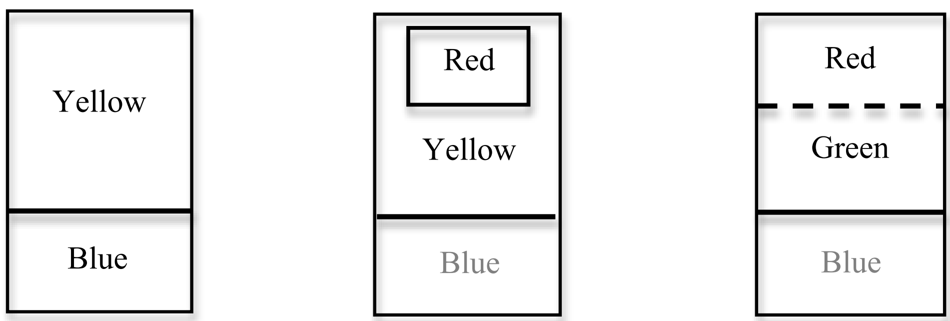
2.1.3 Deriving Color Terms (Jaspers 2011)

The parallelism in the logic and color term hexagons, and the two polar oppositions B – Y and

R – G in the latter, suggests a an approach to “non-natural” color words similar to that deployed for logic.

Suppose the basic domain is color percepts. The initial division in the space is between the (S-cone) Blue and its (L+M) complementary Yellow (25a). Within the residue non-Blue percept space, we can either carve out the subset Red (L), leaving Yellow as the super set space (L+M) (25b), or we can divide the Yellow percept space exhaustively into Red (L) and non-Red (M) (25c):

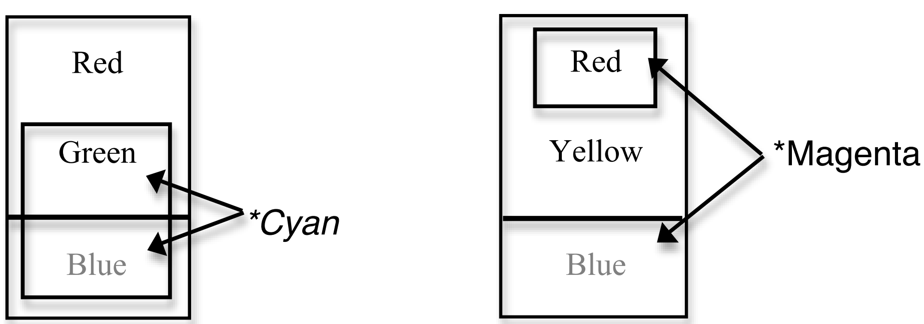
(25) a. *Step 1* b. *Step 2* c. *Step 2’*



Natural color terms match natural divisions of the percept space.

Non-naturalness of **cyan* and **magenta* follows from their percepts’ noncongruency with natural divisions of the percept space; both cut across the B – Y division (26a,b):

(26) a. *Illicit Step* b. *Illicit Step*



Summary:

Human cognition appears to deploy parallel constraints on logical concept formation and color percept formation:

- Observed in the availability of <u>naturally occurring</u> words for certain logic/color percepts, but not in availabity *per se*. Non-natural concepts in the respective domains can be lexified by system external means (*nand, iff, nall* / *magenta*, *cyan*), or by productive, compositional means (*if and only if*, *all or no* / *redish-blue, blue-green*).

- The constraints are on the systematic partioning of an antecedently given conceptual or perceptual space. Divisions are made within existing divisions, and not across them. This accounts for “missing words”.
- The structures for logic concepts and color percepts appear to be isomorphic: the same system is in play in both.

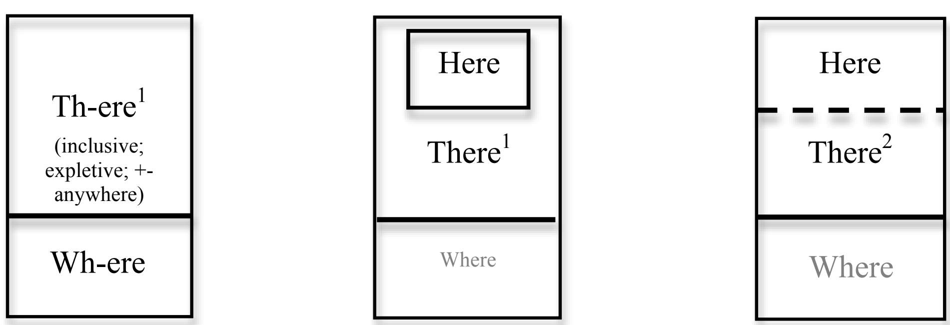
More Missing O-corners (and U-corners)

Extension 1: From logic to deixis
Missing O-corners in the deictic (distal vs proximate) vs. nondeictic realm

*1.1 Spatial deixis: th-ere / wh-ere vs. here / *wh-here*

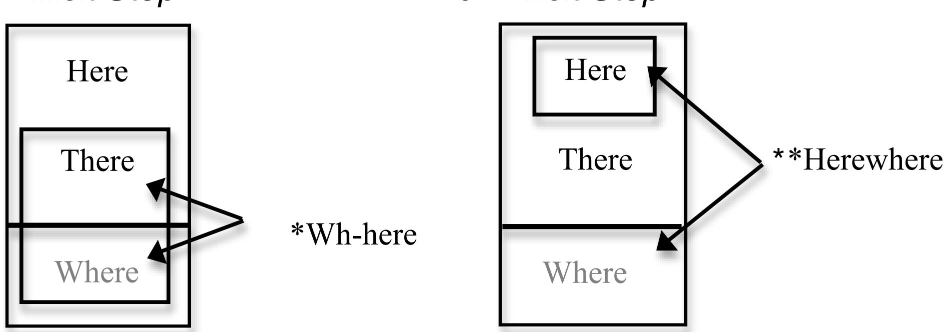
The parallelism in the logic, color and spatial deixis term hexagons, and the two polar oppositions in both, suggests an approach to “non-natural” locational deixis words similar to that deployed for logic and color.
Suppose the basic domain is that of spatial locations. The initial division in the space is between the (nondeictic) *where* and its deictic but nonspecific (High+Middle) complementary *there*^{1} (25a). The latter is inclusive, including the denotation of *here* in its own denotation. Its prototypical instantation is functional-expletive *there*^{1}, which can indeed be used both with *here* and with *there*^{2}, as in *there*^{1} is nobody *here/there*^{2}. Within the residue *there*^{1} space, we can either carve out the subset *here*, leaving *there*^{1} as the super set space (H+M) (25b), or we can divide the *there*^{1} space exhaustively into *here* (H) and *there*^{2} (M) (25c):

(25) a. *Step 1* b. *Step 2* c. *Step 2’*



Natural locational deixis terms match natural divisions of the concept space.
Non-naturalness of **wh-here* and **herewhere* follows from their concepts’ noncongruency with natural divisions of the concept space; both cut across the *there*^{1} – *where* division (26a,b):

(26) a. *Illicit Step* b. *Illicit Step*

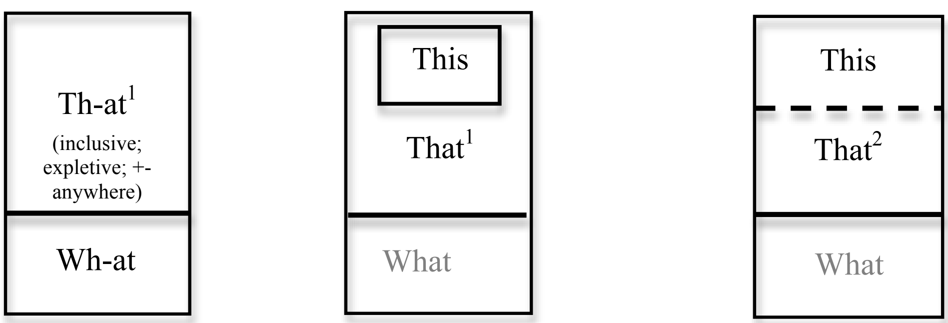


*1.2. The same story for temporal deixis: th-en / wh-en vs. now / *wh-ow*

*1.3. Demonstrative deixis: th-at / wh-at vs. this / *wh-is*

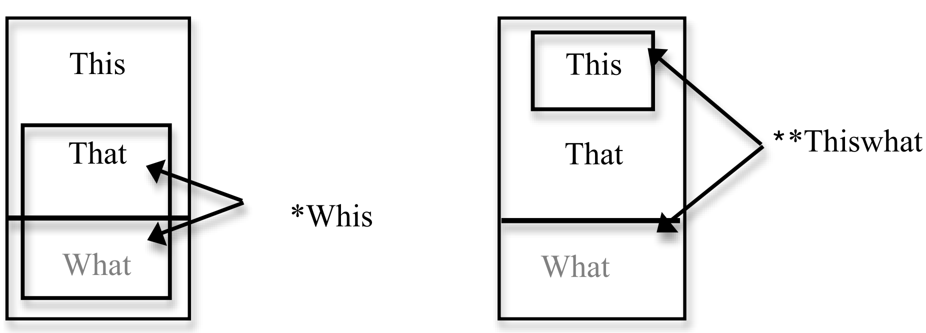
The parallelism in the logic, color, spatial and temporal deixis and demonstrative term hexagons, and the two polar oppositions in both, suggests an approach to "non-natural" demonstrative words similar to that deployed for logic, color and locational deixis.
Suppose the basic domain is "demonstrables". The initial division in the space is between the (indefinite) *what* and its definite but nonspecific (High+Middle) complementary *that*^{1} (25a). The latter is inclusive, including the denotation of *this* in its denotation. Its prototypical instantation is functional *that*^{1}, as in *that*^{1} is a good boy! Within the residue *that*^{1} space, we can either carve out the subset *this*, leaving *that*^{1} as the super set space (H+M) (25b), or we can divide the *that*^{1} space exhaustively into *this* (H) and non-proximate *that*^{2} (M) (25c):

(25) a. *Step 1* b. *Step 2* c. *Step 2'*



Natural demonstrative terms match natural divisions of the concept space.
Non-naturalness of **whis* and **thiswhat* follows from their concepts' noncongruency with natural divisions of the concept space; both cut across the *that*^{1} – *what* division (26a,b):

(26) a. *Illicit Step* b. *Illicit Step*

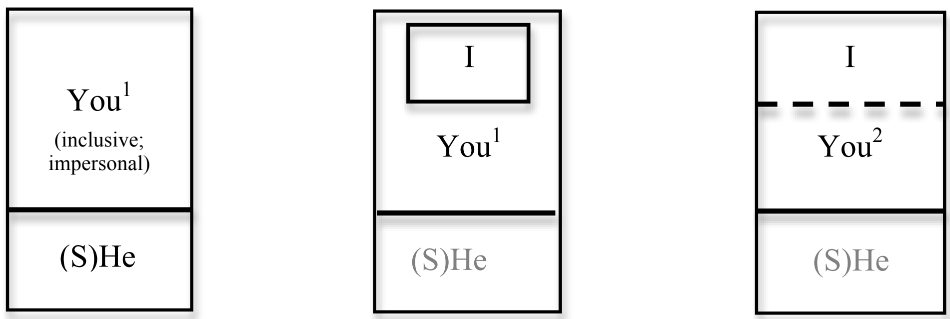


*1.4 person (SINGULAR!) deixis: you / (s)he vs. I/ *(s)h-I*

The parallelism in the logic, color, locational deixis, demonstrative term and personal pronoun hexagons, and the two polar oppositions in both, suggests an approach to "non-natural" singular personal pronouns similar to that deployed for logic, color, locational deixis and

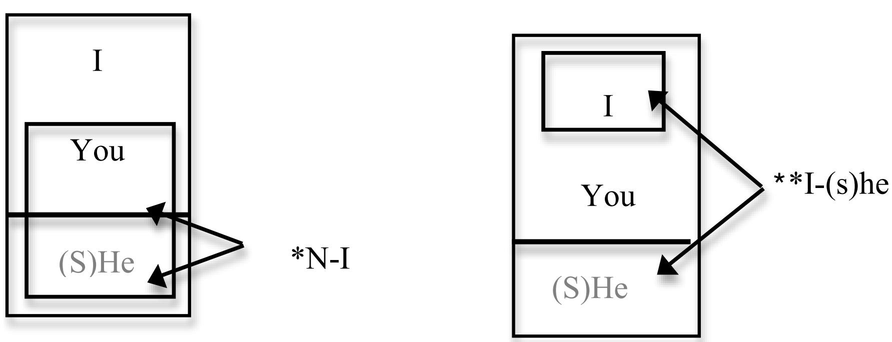
demonstratives.
Suppose the basic domain is "person". The initial division in the space is between the third person *(s)he* and its non-third (High+Middle) complementary "generic" or "impersonal" *you*^{1} (25a). The latter is inclusive, possibly including the denotation of *I* in its denotation. This is clearly shown "in Darja (Arabic as spoken in the Maghreb), where there are two distinct singular second-person pronouns *["nongeneric", "exclusive" or "personal" you*^{2}*; DJ]*, one masculine (used when addressing a man) and one feminine (used when addressing a woman); but when used as generic pronouns, the speaker uses the pronoun with the gender corresponding to his or her own sex, rather than that of the person he or she is addressing."(http://en.wikipedia.org/wiki/Generic_you#cite_note-0) See also http://lughat.blogspot.com/2007/09/impersonal-vs-personal-you.html.
Within the residue *you*^{1} space, we can either carve out the subset *I*, leaving *you*^{1} as the super set space (H+M) (25b), or we can divide the *you*^{1} space exhaustively into *I* (H) and non-inclusive *you*^{2} (M) (25c):

(25) a. *Step 1* b. *Step 2* c. *Step 2'*



Natural demonstrative terms match natural divisions of the concept space.
Non-naturalness of **n-I* (2+3rd person, but semantically SINGULAR, i.e. distributive) and **I-(s)he* (1st+3rd person, but SINGULAR distributive) follows from their concepts' noncongruency with natural divisions of the concept space; both cut across the *you*^{1} – *(s)he* division (26a,b):

(26) a. *Illicit Step* b. *Illicit Step*



Just as in the case of quantifiers, the E-corner (L) stands out in that "there is a fundamental, and ineradicable, difference between the first and second person, on the one hand, and the third person on the other" (Lyons 1977: 638). Evidence for this is that the latter is comparatively rarely marked with a real person-marker, but often alternatively (and therefore not directly within the person-system) "by means of a demonstrative corresponding to the English *this* and *that*" (Siewierska 2004: 5) or "via full nominal expressions" or "no overt expression at all" (Siewierska 2004: 6). All of this is to do with the

fact that “the first and second persons are inherently deictic expressions, that is their interpretation is dependent on the properties of the extralinguistic context of the utterance in which they occur”, the third person can be deictic (or exophoric), but needn’t be (e.g. everybody thinks (s?)he is wise).
(PS: Fourth persons do not really exist. The things often referred to as such are really obviative third persons.)

Final note to 1.1 to 1.4: I found it hard to believe when I noticed it, but I have done a check – as I should, given an hypothesis in my dissertation – and it turns out that the acquisition sequence for EACH of these extensions is the same, namely A, I/Y, E. That is exactly the sequence I postulated for logic in “Operators in the Lexicon”, and it is of course also an important portion of Berlin & Kay’s evolutionary sequence for colors (A: red > Y/I: green/yellow > E: blue)

Extension 2: From logic to mathematics
Missing O-corners in the natural numerals realm

Given that the pattern for the standard predicate calculus is isomorphic to that of the color percept oppositions, the same must be true for a special case of the predicate calculus, namely the one restricted to a domain of two entities (cf. Jaspers 2005):

Both (cardinality *exactly 2*) – either (cardinality *(at least) 1*) – neither (cardinality *0*) - *noth

The cardinalities involved suggest another (and to my taste most important) extension:

A:2	I:≥ 1	Y:(exactly) 1	E: 0 (=<1)	O:*< 2
A:two	I:(at least) one	Y:(exactly) one	E: n-one	O:*ntwo
A:duo	I: unulus/ullus	Y:unulus/ullus	E: n-ullus	O:*nduo

The consequence is inescapable that the *real* basis for the number system cannot be different than that of logic and is cognitive too. Both logic and natural numerals testify to the existence of the same natural system of subtractive concept formation (the NEC-operator sequence of Jaspers (2005)) and are different but isomorphic instantiations thereof. It would have to follow that the only proper way to ground mathematics (number) is to recognize first of all that it is engrained in the constitution of our human mind, just as I have maintained for logic. This link between the two can also make sense of the fact that they are usually taken to be very close to one another. But what to make of our equally strong sense that both are the closest we can get to objective truth? How can that be made compatible with these oppositions being locked into the mind (given that they are homologous to trichromatic color perception)? The only way out seems to me to be the age-old idea that logic and maths “work” and are felt to be good models of extramental reality because the opposition pattern that generates natural logical and mathematical concepts in the human mind is convergent (not necessarily identical) with the algorithm governing “entity architecture” and probably “entity formation/growth” in nature as well. Other is like self and self like other in this respect. This seems to me the only logically conceivable reconciliation between the psychologism I am led to by the facts on the one hand and the factual basis behind positivistic “antipsychologism” on the other, namely the instinctive sense of the existence of objective truth. I have to say “sense of” because reality an sich remains unattainable, but I have no reason to doubt it is there/here, no reason to doubt my reality-an-sich instinct.

Paradoxically, the mentalist perspective drawn here makes sense of the paradox of knowledge: if all concepts are mental creations demonstrably reflecting mental constraints on concept formation, we have to find a mind-internal way to distinguish between such defiled concepts as those denoting entities which can have no extensional correlate (not just "unicorn" but also "London"), and the revered and objective natural logical and natural mathematical ones (and other function words). My claim is that the former are inherently open-ended/class and crucially *disjunctive* concepts, whereas the latter are closed class/exhaustive *conjunctive* concepts (e.g. 3rd person singular means 3rd person AND singular) (See Jaspers 2009). The latter are the functional substrate of language, the CHL-parts of a lexical item that are active in syntactic concatenation. Syntax, so I am then led to claim, must be conjunctive in nature. I see no reason to doubt that this is correct, but will try to work it out (quite a task). Note that if it is, it also follows that one can never introduce a null constituent in the course of a derivation, since conjunctive concatenation would annull all information (the intersection of the null set and any other set is the null set).

In sum: natural/human logic and color percepts, but also maths and deixis: same logic of opposition yielding a sequence of four natural corner lexicalisations in the Blanché star: A > I / Y > E.

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